Don't be intimidated (2)

October 15, 2010

The first question on each paper of the Putnam is often (but not always) not too hard. Do both and you will get 20 points, which is considered a very good score. Remember, you have **3 hours** for each paper. The WWU record is held by David Kennerud, who scored 40 points in 1991.

One of the problems-solving skills you acquire with experience is knowing when to discard an unpromising approach. All the problems below have fairly short solutions, so if your method seems to lead to great complications, try something else. Of course, if you *are* making progress, persevere.

For problems involving natural numbers, e.g., the first four problems below, you can always check special cases. For example, in Problem 2, you could try expressing the numbers 1 to 30 in the given form, in Problem 3, you could try experimenting with various functions $S(N)$, and in Problem 4 you could write out the inequality in full for, say, $m = 3$ and $n = 5$. This will at least give you a feel for the problem, and you might even notice a pattern. Finally, for any problem involving natural numbers, you can always try *mathematical induction.*

1. (Putnam 2008, A2) Alan and Barbara play a game in which they take turns filling entries of an initially empty 2008×2008 array. Alan plays first. At each turn, a player chooses a real number and places it in a vacant entry. The game ends when all the entries are filled. Alan wins if the determinant of the resulting matrix is nonzero; Barbara wins if it is zero. Which player has a winning strategy?

2. (Putnam 2005, A1) Show that every positive integer is a sum of one or more numbers of the form $2^r 3^s$, where r and s are nonnegative integers and no summand divides another. (For example, 23=9+8+6.)

3. (Putnam 2004, A1) Basketball star Shanille O'Keal's team statistician keeps track of the number, $S(N)$, of successful free throws she has made in her first N attempts of the season. Early in the season, $S(N)$ was less than 80% of N, but by the end of the season, $S(N)$ was more than 80% of N. Was there necessarily a moment in between when $S(N)$ was exactly 80% of N?

4. (Putnam 2004, B2) Let m and n be positive integers. Show that

$$\frac{(m+n)!}{(m+n)^{m+n}} < \frac{m!}{m^m}\frac{n!}{n^n}.$$

5. (Putnam 2001, A1) Consider a set S and a binary operation $*$, i.e., for each $a, b \in S, a * b \in S$. Assume $(a * b) * a = b$ for all $a, b \in S$. Prove that $a * (b * a) = b$ for all $a, b \in S$.

6. (Putnam 1988, A1) Let R be the region consisting of the points (x, y) of the cartesian plane satisfying both $|x| - |y| \leq 1$ and $|y| \leq 1$. Sketch the region R and find its area.

Homework

Do all the questions we didn't do in class.