

Don't be intimidated

October 21, 2009

The first question on each paper of the Putnam is often (but not always) not too hard. Do both and you will get 20 points, which is considered a very good score. Remember, you have **3 hours** for each paper. The WWU record is held by David Kennerud, who scored 40 points in 1991.

In the following, A1 is the first question on the morning paper, and B1 is the first question on the afternoon paper.

1. (Putnam 2008, A1) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a function such that $f(x, y) + f(y, z) + f(z, x) = 0$ for all real numbers x, y , and z . Prove that there exists a function $g : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x, y) = g(x) - g(y)$ for all real numbers x and y .

2. (Putnam 2008, B1) What is the maximum number of rational points that can lie on a circle in \mathbb{R}^2 whose center is not a rational point? (A *rational point* is a point both of whose coordinates are rational numbers.)

3. (Putnam 2007, A1) Find all values of α for which the curves $y = \alpha x^2 + \alpha x + \frac{1}{24}$ and $x = \alpha y^2 + \alpha y + \frac{1}{24}$ are tangent to each other.

4. (Putnam 2007, B1) Let f be a polynomial with positive integer coefficients. Prove that if n is a positive integer, then $f(n)$ divides $f(f(n) + 1)$ if and only if $n = 1$. [Note: one must assume f is nonconstant.]

5. (Putnam 2006, A1) Find the volume of the region of points (x, y, z) such that

$$(x^2 + y^2 + z^2 + 8)^2 \leq 36(x^2 + y^2).$$

6. (Putnam 2005, B1) Find a nonzero polynomial $P(x, y)$ such that $P(\lfloor a \rfloor, \lfloor 2a \rfloor) = 0$ for all real numbers a . (Note: $\lfloor v \rfloor$ is the greatest integer less than or equal to v .)
7. (Putnam 2002, A1) Let k be a fixed positive integer. The n^{th} derivative of $\frac{1}{x^k-1}$ has the form $\frac{P_n(x)}{(x^k-1)^{n+1}}$ where $P_n(x)$ is a polynomial. Find $P_n(1)$.
8. (Putnam 1999, A1) Find all polynomials $f(x), g(x)$, and $h(x)$, if they exist, such that for all x ,

$$|f(x)| - |g(x)| + h(x) = \begin{cases} -1 & \text{if } x < -1 \\ 3x + 2 & \text{if } -1 \leq x \leq 0 \\ -2x + 2 & \text{if } x > 0. \end{cases}$$

9. (Putnam 1998, A1) A right circular cone has base of radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube?
10. (Putnam 1988, A1) A *composite* (positive integer) is a product ab with a and b not necessarily distinct integers in $\{2, 3, 4, \dots\}$. Show that every composite is expressible as $xy + xz + yz + 1$, with x, y , and z positive integers.

Homework

Do all the questions we didn't do in class.