Math 302 Introduction to Proofs via Number Theory Spring 2020

Instructor Dr. Amites Sarkar

Text There is no textbook for this course.

Preface

This course will take place online through Canvas and Zoom. My priorities are to make the course accessible, and the grading fair. The ideal way to participate is synchronously, at the regularly scheduled meeting time of 1pm on Mondays, Tuesdays, Thursdays and Fridays (I will host a Zoom meeting and invite everyone on the class list). However, I intend to record each class meeting, and make the recordings available through Canvas, so it should be possible to complete the class asynchronously, if need be. The grading will be based on homework, presentations (through Canvas and Zoom), a midterm, and a final; see below for details.

Overview

Number theory, the "queen of mathematics", has a long and rich history. In this course we will encounter some fundamental ideas of **Euclid** (who lived around 2300 years ago), as well more recent contributions from great mathematicians such as **Fermat**, **Euler** and **Gauss**. Fascinating work in number theory is still being done today, since there are many important unsolved problems. These include the **twin prime conjecture** (are there infinitely many pairs of primes which differ by exactly 2), **Goldbach's conjecture** (is every even number greater than 2 the sum of two primes) and the **Riemann hypothesis** (which is a bit harder to explain). Number theory also provides the mathematical basis for modern **cryptography**, some of which we shall study in this course.

However, the main course objective is to introduce you to mathematical **proof**. Moreover, you will learn about proofs not by reading them but by **inventing them yourself**. This will, for most of you, be unlike any other mathematics course you have taken. It will involve investigating on your own, forming and testing hypotheses, and then trying to prove what you believe to be true. This is the true essence of mathematics.

Exams

Midterm Friday 8 May
Final Monday 8 June 1–3 pm

Homework

There will be written homework assignments, based on the eight theorem sheets, which will be posted online at regular intervals throughout the quarter. You should send me your solutions through Canvas. I will provide feedback and evaluation of your proofs based on both the mathematical content and the writing style. We will talk more about how to write proofs in the first week of class.

Grading

Your grade for the course will be based as follows:

Written proofs and Zoom presentations	50%
Midterm	20%
Final	30%

The exams will be held online: I am still working out the details. If you feel too ill to take an exam, don't take it, but let me know and I will make arrangements.

Office hours

Through Zoom, by appointment. My e-mail is amites.sarkar@wwu.edu

Course objectives

The successful student will demonstrate:

1. Proficiency in writing and presenting clear, complete and correct mathematical proofs.

2. An understanding of the principles of mathematical induction.

3. An understanding of the properties and principles of divisibility, including the Euclidean algorithm.

4. Knowledge of the proof of the Fundamental Theorem of Arithmetic, and its applications.

5. Knowledge of basic facts about the prime numbers, and the proof of the infinitude of primes.

6. Competence in congruence arithmetic and use of the Chinese remainder theorem.

7. Knowledge of the proofs of Fermat's and Wilson's Theorems and their applications.

Sources of help

Please let me know about any questions or concerns you may have about the course. You are welcome to discuss material (e.g. online) with other students in class. However, if you have had substantial assistance in doing a problem, please indicate this in your written solution. You are not allowed to look at textbooks or the Internet for solutions to problems or proofs of theorems. That would contradict the spirit of the course.